GCSE Maths – Algebra

Equivalent Algebraic Expressions

Notes

WORKSHEET

Equivalent Algebraic Expressions

Expressions

An expression is a **group of terms** related to each other using mathematical operations. For example, $4x + 2xy$ is an expression, as is $x^2 + x$.

Equations

An equation is a statement with an equals sign which states that **two expressions are equal**. For example, $4x + 2xy = x^2 + x$ or $4x^2 + x + 5 = 0$.

Identities

An identity is an **equation** that is **true no matter what values are inputted**. Examples include $4x + 2x = 6x$ and $y + y + y = 3y$. The equals sign, $=$, can be replaced with the 'identical to' sign, $\equiv$, when dealing with identities.

Example: Categorise the following into expressions, equations, and identities.

a) $x^2 + 2x^2 = 3x^2$	b) $4a(5a) \equiv 20a^2$	c) $76xy^{38}$
d) $\sqrt{x} = y^2$	e) $x^2 + 2x + 5 = 74$	f) $(x + y)^2$

- *For a) both sides of the equals sign are the same, regardless of what value x takes, meaning it is an **identity**.*
- *For b) we also have an **identity** as both expressions on either side of the equals sign are there same for any value of a which is input. Note the use of the 'identical to' sign.*
- *c) is not set equal to anything meaning it is an **expression**.*
- *d) is two expressions equalling each other and does not hold for all values of x and y, so it is an **equation**.*
- *Similarly, e) is an **equation** because it is not true for all values of x. For example, if we substitute $x = 1$ into the left-hand side we will not get 74. This shows it is not an identity.*
- *f) is an **expression** for the same reasons c) is.*

To summarise:

***Expressions**: c) and f)*

***Equations**: d) and e)*

***Identities**: b) and a)*

Mathematical Proofs (Higher Only)

In mathematics, a proof is a **sequence of true statements** that logically follow each other to **prove** a required result. An algebraic proof uses algebra instead of numbers, which means we can prove things are true for **all numbers at once**.

The following examples show how to structure proofs.

Example: Prove the product of two even numbers is always even.

Let n and m be two different integers. Then $2n$ and $2m$ are both even as they both clearly have a factor of 2. The product can be written as

$$2n \times 2m = 4nm = 2(2nm)$$

*Since $2(2nm)$ has a **factor** of 2, it is **even**. This completes the proof: we never specified what n and m are, so it holds for all even numbers (as they can be constructed by choosing an appropriate n or m and multiplying by 2).*

Example: Prove the product of an even and odd number is always even.

Let n and m be different integers. Then, $2n$ is even and $(2m + 1)$ is odd since it is one more than the even number $2m$. The product can be written as

$$2n \times (2m + 1) = 4nm + 2n = 2(2nm + n)$$

The product has a factor of 2, and hence is even.

Equivalent Algebraic Expressions – Practice Questions

1. Categorise the following into expressions, equations, and identities:

   i) $x + y + z$
   ii) $x + y + z = 64$
   iii) $3x + 2x - 4z + 4z^2 = 5x + 4(z^2 - z)$
   iv) $7x^2 + 2x + 2021 = 0$

2. If the area of a rectangle is 50, and the sides are labelled with x and y:

   i) Write an equation for the area
   ii) Write an expression for the perimeter

3. Prove that the square of an even number is always even (Higher Only)

4. Prove that the sum of 3 consecutive numbers is always divisible by 3 (Higher Only)

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.